Task-Space Inverse Dynamics: Implementation

Optimization-based Robot Control

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Introduction

Key Concepts

Task

- Motion
- Force
- Actuation

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- translates them into HQP

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HQP Solver

- solves a HQP

Other Concepts

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- purely mathematical
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- provides utility functions to compute robot quantities
- e.g., mass matrix, Jacobians

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Trajectory

- maps time to vector values
- pos, vel, acc
- position and velocity can have different sizes (Lie groups)

Details

ConstraintBase

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Three kinds of constraints:

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- Equalities, represented by matrix A and vector a:

$$Ax = a$$

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$$lb \leq Ax \leq ub$$

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- Equalities, represented by matrix A and vector a:

$$Ax = a$$

- Inequalities, represented by matrix A and vectors lb and ub:

$$lb \leq Ax \leq ub$$

- Bounds, represented by vectors lb and ub:

$$lb \leq x \leq ub$$

ConstraintBase

```
ConstraintBase(string name, int rows, int cols);

bool isEquality();
bool isInequality();
bool isBound();

Matrix matrix();
Vector vector();
Vector lowerBound();
Vector upperBound();

bool setMatrix(Matrix A);
bool setVector(Vector b);
bool setLowerBound(Vector lb);
bool setUpperBound(Vector ub);

bool checkConstraint(Vector x);
```

TaskBase

Interface of `TaskBase`:

```
TaskBase(string name, Model model);
Constraint compute(double t, Vector q, Vector v, Data data);
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- `TaskContactForce`: linear function of contact forces
- `TaskActuation`: linear function of joint torques

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Tasks can compute either:

- equality constraints, e.g., `TaskComEquality`, `TaskJointPosture`, `TaskSE3Equality`

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- bounds, e.g., `TaskJointBounds`
- inequality constraints, e.g., friction cones

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```
ContactBase(name, Kp, Kd, bodyName, regWeight);
ConstraintBase computeMotionTask(t, q, v, data);
InequalityConstraint computeForceTask(t, q, v, data);
ConstraintBase computeForceRegularizationTask(t, q, v, data);
Matrix computeForceGeneratorMatrix();
```

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Motion task:

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- $J\dot{v} = -\dot{J}v$

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Force task:

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- e.g., friction cone constraints
- $Af \leq a$

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- e.g., keep them close to friction cone center

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Force-Generator matrix T:

- maps force variables to motion constraint representation
- Dynamic: $M\dot{v} + h = S^\top \tau + J^\top T f$
- Motion constraint: $J\dot{v} = -\dot{J}v$
- Friction cones: $Af \leq a$

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- unilateral plane contact $\rightarrow$ 6d motion constraint
- minimal force representation $\rightarrow$ 6d (3d force + 3d moment)

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SOLUTION

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SOLUTION

- use 6d representation for motion constraint $J\dot{v} = -\dot{J}v \in \mathbb{R}^6$
- but 12d representation for force variable $f \in \mathbb{R}^{12}$
- force-generator matrix $T \in \mathbb{R}^{6\times 12}$ defines mapping between two representations: $\tau_{contact} = J^\top Tf$

InverseDynamicsFormulationBase

Central class of the whole library

Methods to add tasks:

```
addMotionTask(MotionTask task, double weight, int priority);
addForceTask(ForceTask task, double weight, int priority);
addTorqueTask(TorqueTask task, double weight, int priority);
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HqpData computeProblemData(double time, Vector q, Vector v);
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Methods to convert TSID problem into (Hierarchical) QP:

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HqpData computeProblemData(double time, Vector q, Vector v);
```

HqpData defined as:

```
#typedef vector<pair<double, ConstraintBase>> ConstraintLevel
#typedef vector<ConstraintLevel> HqpData
```

Python Example

Ex 1 UR5

- Robot manipulator

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- end-effector control

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- torque limits

Ex 1 UR5

- Robot manipulator
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- joint velocity limits

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- Code snippets
- Biped robot with both feet on the ground (double support)
- Control center of mass (CoM) for balance
- Control joint angles (posture) for whole-body stability
- Good starting point before moving to more complex scenarios

Create Robot Wrapper

```
import pinocchio as se3
from tsid import RobotWrapper, ...

...
robot = RobotWrapper(urdf, vec, se3.JointModelFreeFlyer(),
                     False)
```

Create Inverse Dynamics Formulation

```
invdyn = InverseDynamicsFormulationAccForce("tsid", robot,
                                             False)

q = ...
v = ...
invdyn.computeProblemData(t, q, v)
```

Create Contact

```
contactRF = Contact6d("contact_rfoot", robot, rf_frame_name,
                      contact_points, contact_normal, mu,
                      fMin, fMax, w_forceReg)

contactRF.setKp(...)
contactRF.setKd(...)

H_rf_ref = ...
contactRF.setReference(H_rf_ref)

invdyn.addRigidContact(contactRF)

# repeat for other contact(s)
```

Create Center-of-Mass Task

```
comTask = TaskComEquality("task-com", robot)

comTask.setKp(...)
comTask.setKd(...)

invdyn.addMotionTask(comTask, w_com, 1, 0.0)
```

Create Posture Task

```
postureTask = TaskJointPosture("task-posture", robot)

postureTask.setKp(...)
postureTask.setKd(...)

invdyn.addMotionTask(postureTask, w_posture, 1, 0.0)
```

Create Reference Task Trajectories

```
com_ref = robot.com(data)
trajCom = TrajectoryEuclidianConstant("traj_com", com_ref)

q_ref = q[7:]
trajPosture = TrajectoryEuclidianConstant("traj_joint", q_ref)
```

Create HQP Solver

```
solver = SolverHQuadProg("qp solver")
solver.resize(invdyn.nVar, invdyn.nEq, invdyn.nIn)
```

Control Loop

```
for i in range(0, N_SIMULATION_STEPS):
    comTask.setReference(trajCom.computeNext())
    postureTask.setReference(trajPosture.computeNext())

    # get current state estimation
    (q, v) = ...

    HQPData = invdyn.computeProblemData(t, q, v)

    sol = solver.solve(HQPData)
    tau = invdyn.getActuatorForces(sol)

    # send desired joint torques (tau) to actuators
    ...
```

Simulation Loop

```
for i in range(0, N_SIMULATION_STEPS):
    ...

    # assuming perfect torque-acceleration tracking...
    dv = invdyn.getAccelerations(sol)

    # integrate desired accelerations
    q = se3.integrate(robot.model(), q, dt*v)
    v += dt*dv

    # increase time
    t += dt
```

Exercises

Exercise 2: CoM Sinusoidal Tracking

Run provided code (`tsid/exercizes/ex_2.py`) and check the sinusoidal reference CoM tracking

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- Set reference CoM outside support polygon (e.g., 20 cm to the side), what happens? Why?

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- Increase CoM frequency until tracking gets bad. Why does that happen?

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- Change CoM/posture weights and see effect
- Set reference CoM outside support polygon (e.g., 20 cm to the side), what happens? Why?
- Increase CoM frequency until tracking gets bad. Why does that happen?
- Add contact on hand

Exercise 3: Balancing

Run provided code
(`tsid/exercizes/ex_3_biped_balance_with_gui.py`)

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- Push robot and check reaction

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- Move reference CoM position
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Run provided code
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- Break contact with right foot

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- Break contact with right foot
- Move reference right foot