Robust Task-Space Inverse Dynamics

Mathematical Details

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Introduction

These slides explain the mathematical details of the robust optimization problems solved in "Robustness to Joint-Torque Tracking Errors in Task-Space Inverse Dynamics" [1].

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Worst-Case Robust Least-Squares

Uncertainty Model

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- $e^{max} \in \mathbb{R}^n$ is maximum torque tracking error

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- problem not tractable in this form because of infinite number of constraints
- beware of potential infeasibility: there may be no x satisfying constraints for any e

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- Geometric interpretation: do not check inequality for all values of U, but only for worst corner

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- Reformulate Robust LS

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- where $|B|$ contains absolute values of elements of B

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- If necessary set $s = 0 \rightarrow$ standard constraints

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- Handle infeasibility by introducing slack variable

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- Decoupled covariance matrix $\Sigma = \text{diag}(\begin{bmatrix} \sigma_1^2 & \dots & \sigma_n^2 \end{bmatrix})$

Stochastic Least-Squares Program

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- $p(.)$ not convex (in general) $\rightarrow$ not wise to use it directly!

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- Alternative: no trade off $\rightarrow$ apply strict prioritization approach!

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- evaluate m univariate CDFs rather than one multivariate CDF $\rightarrow$ much faster!

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- Final robust problem:

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- Final problem is nonlinear, convex and smooth

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